Combustion of Fuel Droplets

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Droplets in the size range commonly encountered in spray combustors (50 to 200 μ) were produced in dilute sprays with a narrow size-distribution by means of an electrostatic atomizer. They were allowed to fall through a flat premixed methane-oxygen-nitrogen flame. As the droplets entered the flame, they ignited and subsequently burned in the hot gases resulting from the combustion of methane. The composition and temperature of these gases were controlled by adjusting the composition of the preflame gaseous mixture. The point of droplet ignition and the droplet lifetime under various experimental conditions were measured by optical methods. Such measurements were made with a variety of fuels. In the presence of sufficient oxygen to sustain combustion, it was found that ignition occurred in less than about 0.1 msec. However, the burning lifetime of the droplets varied from 2 to 20 msec depending on the initial droplet diameter and the gaseous environment. In the absence of sufficient oxygen for complete combustion, both evaporation of the droplet and thermal cracking of the fuel vapor were noted. The experimental data can be interpreted in terms of a model in which the rate of conductive heat transfer to the droplet controls its burning rate under specified conditions of oxidizer and fuel composition.

Introduction

BASED on experimental investigations,[1] the burning rate of a single fuel drop may be described by a steady-state model in which the chemical conversion of the fuel to combustion products occurs in a narrow region surrounding the drop at a finite distance from its surface. The rate of vaporization of the fuel from the droplet surface is governed by the rate of transfer of heat from the combustion zone back to the droplet surface. The primary modes of heat transfer are conduction and convection, but the contribution of convective heat transfer to the combustion processes is in some respects negligible where convective forces on the droplet are small. Such is the case for very small droplets ($<500\ \mu$) or for those burning in a low-pressure atmosphere[2] ($p < 100$ mm Hg).

The experimental work[1] on which these conclusions are based consists almost exclusively of studies of large drops ($>1000\ \mu$) burning singly in controlled atmospheres. It has been assumed that this heat-transfer-controlled combustion mechanism applies also to drops in the size range commonly encountered in spray combustors (10 to 200 μ), but there has been no direct experimental verification of this. Also, there has been no fundamental quantitative study of the effects of oxidizer composition, droplet velocity, or ambient temperature on the combustion of individual fuel drops in this size range. This paper describes the results of an experimental study of such systems.

Experiments

The apparatus employed in this study is illustrated in Fig. 1. Monodisperse, dilute sprays of hydrocarbon fuels were produced in an atomizer based on the electrostatic principle.[3] The droplets, thus produced, were ignited by allowing them to fall through a flame of simplified geometry supported by a mixture of methane, air, and oxygen or nitrogen. This ignition flame was established at the outlet of an inverted, water-cooled Bartholomé nozzle[4] and was anchored on a straight wire so that it assumed the shape of a broad, flat-sided tent. Viewed end-on, the ignition flame had the appearance of an inverted V; the monodisperse spray of droplets, falling through the nozzle, pierced the flame and continued their freefall while burning (Fig. 2).

The nature of the atmosphere surrounding the burning droplets was governed by the composition of the premixed gases leaving the flame envelope. The temperature and spatial persistence of this stream of gas were evaluated by introducing a small quantity of sodium chloride vapor into the preflame gas and measuring the *D*-line reversal temperature[5] at various points in the postflame region. The value varied slightly with oxygen content of the system around a mean of 1775°K. By adding submicron-size particles of talcum powder to the preflame gas and photographing the postflame field with stroboscopic illumination, the gas velocity and flow patterns were evaluated. The length of this region was sufficient to contain all but the largest drops for their entire burning lifetimes. Hence, the oxygen content of the atmosphere surrounding the burning drops was conveniently varied by adding to the premixed gases oxygen in excess of that required to consume the methane. In this fashion the residual atmosphere below the ignition flame was adjusted from no oxygen to 80 wt % oxygen.

The velocities of the falling droplets were measured by photographing them under stroboscopic illumination. The repetition rate of the stroboscopic lamp was calibrated (400 $\sec^{-1}$, flash duration 1 μsec) so that the distance between successive droplet images on the film gave a direct measure of the droplet's velocity (Fig. 2). The total length of the trajectory of the burning droplet also was measured from the photograph, and from these parameters the burning lifetime τ was calculated.

A magnesium oxide replication technique[6] was employed to measure the initial diameter of droplets in the unignited spray.

Based on such measurements of burning lifetime τ and initial droplet diameter D_0, the average burning constants ϵ were calculated from the relation

$$\epsilon = D_0^2/\tau \tag{1}$$

This equation is based on the observation that, for a burning droplet, the change in droplet area is a linear function of time[7-9] so that $D^2 = D_0^2 - \epsilon t$. The instantaneous mass burning rate of the droplet, m, is related to ϵ by the equation[8]

$$m = \pi \rho D \epsilon / 4 \tag{2}$$

where ρ is the density of the liquid fuel.

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This treatment of the data obtained in the experiments was valid for evaluation of the burning constants because the duration of the droplet preheating period, or the ignition lag, appeared to be very short in relation to the total burning lifetime of the droplet. In the presence of sufficient oxygen, all droplets were observed to ignite during their traverse through the visible zone of the methane-air flame. At the droplet velocities experienced in the apparatus, the time of this traverse was of the order of 0.1 msec. The total burning lifetimes of the droplets encountered in the study were in the range 2 to 20 msec.

Hexadecane was used to assess the burning characteristics of single component droplets with initial diameters in the range 80 to 200 μ. Samples of no. 2 heating oil from three different refiners and a sample of kerosene were used to investigate the burning characteristics of distillate fuel droplets. A sample of no. 2 heating oil decolorized over activated charcoal and a sample of hexadecane artificially colored with the addition of a small quantity of Apiezon W black wax also were burned in selected experiments.

Results

The experimental values of droplet burning constants obtained in the study are summarized in Table 1. Additional data from experiments in which the velocity of the gases in the postflame region was evaluated are shown in Table 2.

In addition to providing burning rate data, selected burning droplet photographs were used to evaluate the width of the combustion zone associated with a droplet during its burning lifetime. For the purpose of analysis, this width was assumed to be proportional to combustion zone radius r_c. By combining this information with the values of droplet diameter computed from the measured average burning constant at various points in the droplet's trajectory, values of the ratio of combustion zone radius to droplet radius r_c/r_l as a function of the fraction of expended combustion lifetime t/τ were obtained. This variation of r_c/r_l for droplets of hexadecane burning in various oxidizer compositions is shown in Fig. 3.

Discussion

Burning Rates

The size range[2] and relative velocities[10] (Table 2) of the droplets observed in this study indicated that combustion occurred in a region in which convective forces had little effect upon burning rate. Hence, the experimental values of the burning constant were interpreted in terms of a steady-state model in which conductive heat transfer from the combustion zone to the droplet surface is the rate-controlling process.[8]

In such a model the burning constant ϵ is expressed as a function of various thermodynamic and transport parameters appropriate to the system:

$$\epsilon = \frac{8\lambda}{\rho C} \ln\left[\frac{q}{L}\frac{Y}{i} + 1 - \frac{C}{L}(T_l - T_\infty)\right] \tag{3}$$

where

λ = thermal conductivity of the ambient gas, cal/sec-cm-°K
C = specific heat of the ambient gas, cal/g-°K
q = heat of combustion of the fuel, cal/g
L = heat of vaporization of the fuel, cal/g
i = stoichiometric oxygen-fuel weight ratio
Y = weight-fraction of oxygen in the ambient gas
T_l = temperature at the droplet surface, °K
T_∞ = temperature of the ambient gas, °K

The experimentally determined variation of burning constant with oxidizer composition for droplets of hexadecane and no. 2 heating oil is shown in Fig. 4. A theoretical curve (broken line) calculated from Eq. (3) with thermodynamic and transport parameters selected from those appropriate to the hexadecane-air system[2] is included also. The multicomponent nature of the heating oil makes difficult a proper selection of such physical parameters for burning droplets of this fuel. Hence the experimental data cannot be correlated

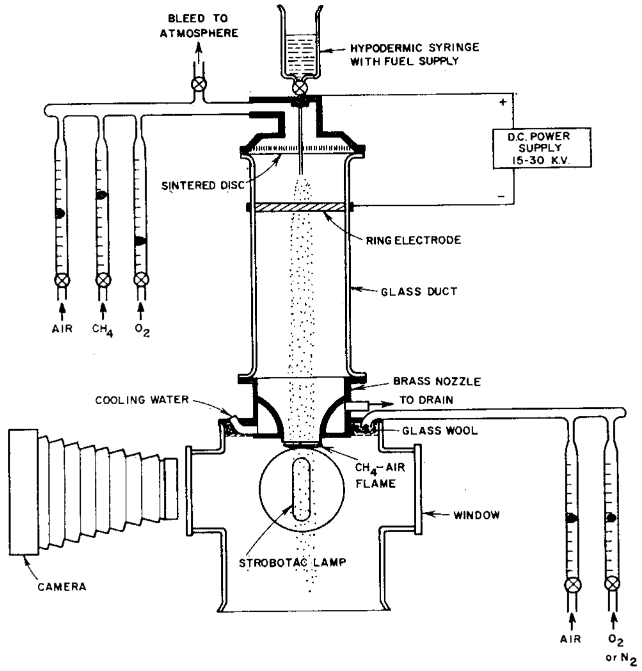

Fig. 1 Apparatus for producing droplets, igniting them, and measuring their burning rates

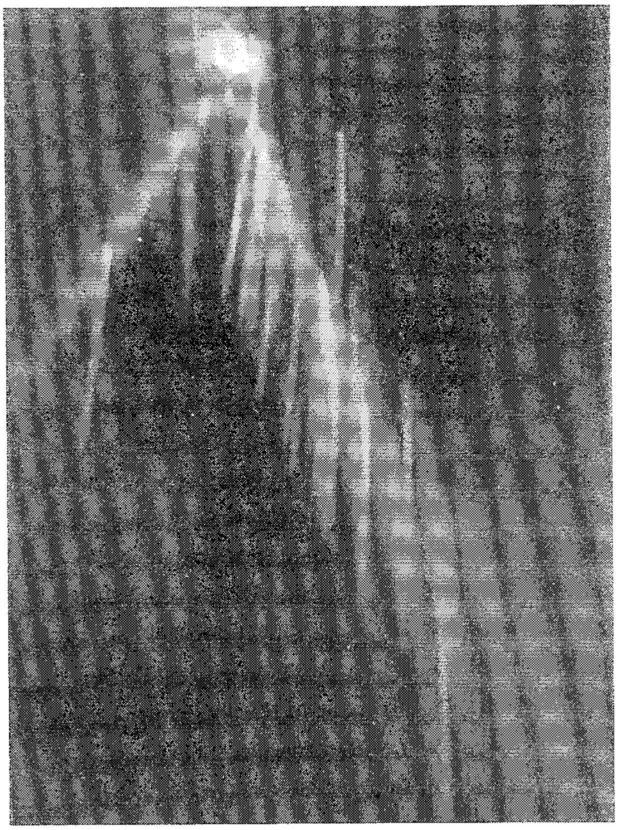

Fig. 2 Photograph of stroboscope-illuminated burning droplets and associated light streaks used to evaluate burning lifetime τ (ignition flame is tent-shaped, because it is anchored to a wire oriented perpendicular to the plane of the photograph near the mouth of the burner nozzle)

Table 1 Experimental values of droplet burning constants. $T_\infty \cong 1775°K$

Fuel	Wt fraction of oxygen in atmosphere, Y	Initial diameter D_0, μ	Burning constant[a] ϵ, cm^2/sec $\times 10^3$
Hexadecane	0	86	3
	0.047	131	5.8
	0.065	128	6.5
	0.116	192	9.6
	0.122	96	8.1
	0.203	166	9.9
	0.24	147	11
	0.42	96	13
		145	12
		153	13
	0.81	115	17
		197	18
No. 2 heating oil	0	97	5
Refiner A	0.05	50	4.6
		80	5.9
		85	8.3
		141	6.5
	0.12	73	7.4
		86	7.6
	0.133	61	5.4
	0.240	53	6.6
		103	7.9
		129	8.5
		141	9.5
	0.269	51	5.5
		61	6.7
		79	7.9
		84	7.6
		102	6.5
	0.42	53	7.4
		61	7.9
		72	7.3
		78	7.7
		100	11.2
		119	9.7
		123	10.4
		141	10.3
		153	12.7
	0.71	116	12.2
Refiner B	0.23	113	9.3
	0.42	86	13.7
Refiner C	0.22	100	9.1
		110	11.5
Kerosene	0.22	74	8.8
		92	9.5
Decolorized no. 2 heating oil Refiner A	0.23	74	8.1
		89	9.5
		102	7.3
		104	8.6
		132	7.6
Amber-colored hexadecane	0.23	77	8.2
		85	5.4
		90	9.2
		98	10.8
		112	9.5
		117	10.9
		130	9.0
		134	8.9
		138	8.8
		154	14

[a] Obviously, a droplet in an atmosphere containing no oxygen cannot be said to be burning. The gas temperatures experienced by the droplets in the experiments, however, caused rapid vaporization of the fuel accompanied by visible radiation even when the gas contained little or no oxygen. In these cases Eq. (1) is still valid, but the parameter ϵ might be termed more aptly the "evaporation constant." For the sake of uniformity and simplicity, however, the term "burning constant" is employed throughout this paper to refer to the parameter ϵ.

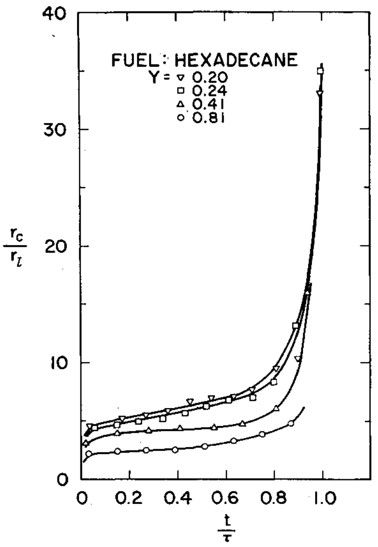

Fig. 3 Variation in r_c/r_l during the lifetimes of hexadecane droplets burning in various oxidizer compositions

quantitatively with a theoretically derived curve. However, the general functional dependence of droplet burning constant on oxidizer composition suggests that the burning rate of droplets in the size range of interest in spray combustion is determined substantially by conductive heat transfer. In the case of hexadecane, the values of burning constant of droplets in the 80 to 200-μ-diameter range are consistent with values obtained in the absence of natural convection with drops possessing initial diameters greater than 1000 μ.[2]

Radiative Heat Transfer

Under some circumstances the absorption of radiation by a fuel droplet may contribute significantly to the rate of heat transfer from the combustion zone to the droplet. Most of the radiation by the hot flame gas, resulting from the combustion of low molecular weight hydrocarbons such as methane, is associated with carbon dioxide and occurs near

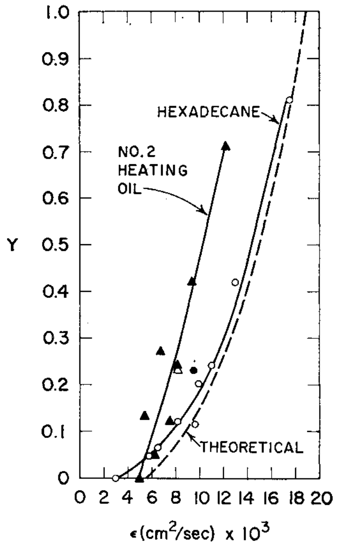

Fig. 4 Dependence of droplet burning constant ϵ on the oxygen content of the atmosphere Y; the broken line represents the dependence predicted by theory [Eq. (3)] for a droplet-oxidizer system with the following characteristics: $T_\infty = 1500°K$ (2277°F), $T_l = 520°K$ (476°F), $C = 0.271$ cal/g-°K, $\rho = 0.675$ g/cm^3, $q = 10{,}600$ cal/g, $L = 225$ cal/g, $i = 3.46$, $\lambda = 5.78 \times 10^{-4}$ cal/sec-cm-°K. The experimental points are averaged values of the burning constants taken from Table 1 (○ pure hexadecane, ● colored hexadecane, ▲ no. 2 heating oil, △ decolorized no. 2 heating oil)

Table 2 Effect of droplet velocity on burning constant. $Y = 0.42$

Fuel	Initial diameter D_0, μ	Droplet velocity, cm/sec	Gas velocity, cm/sec	Burning constant ϵ, cm²/sec × 10³	Reynolds number[a]
Hexadecane	96	110	105	13	0.015
	153	100	80	13	0.38
No. 2 heating oil	141	125	105	10	0.088
	141	180	105	10	0.33

[a] Based on velocity of drop relative to air at 1775°K.

a wavelength of about 4.5 μ.[11] Consequently, the fuel droplet must absorb strongly in that region of the spectrum if radiative heat transfer is to be important. An infrared absorption spectrum of a sample of no. 2 heating oil showed very little absorption near 4.5 μ, and no significant diminution in the burning constant of the decolorized heating oil droplets was observed (Table 1). Indeed, even droplets of the hexadecane sample that was colored with a colloidal black dye (Apiezon "W") showed no increase in burning constant over those of the uncolored fuel (Table 1). Hence, it is concluded that radiative heat transfer is unimportant in these experiments. The spectrum of radiation emitted by the hot walls of a combustion chamber, however, may be different from that of the droplet flame, and in such an environment radiative heat transfer may contribute to the droplet burning rate.

Flame Size

The fact that the burning constant of these droplets can be demonstrated to depend primarily on the rate of conductive heat transfer between combustion zone and droplet surface does not mean that convective forces play no role in the combustion process. Other investigators[2, 12] have shown that, although droplet burning rate is a relatively insensitive function of the magnitude of the convective force, other features of the combustion process are affected radically by convection. One of these which has been studied widely is flame shape and size in relation to the liquid droplet.[12] According to the model on which Eq. (3) is based,[8] the ratio of combustion zone radius and droplet radius, r_c/r_l, should be constant during the lifetime of the drop and should be related to the oxygen content of atmosphere by the equation

$$\frac{r_c}{r_l} = \frac{\ln[(q/L)(Y/i) + 1 - (C/L)(T_l - T_\infty)]}{\ln[1 + (Y/i)]} \tag{4}$$

As one might expect from a model that ignores convection, the values of r_c/r_l calculated from Eq. (4) differ substantially from those measured in these experiments. However, the functional dependence of r_c/r_l on Y predicted by the model appears to apply over the range of diameters (Table 3). The experimental values of r_c/r_l used in Table 3 were derived from the data presented in Fig. 3 by calculating, from the burning constants, the instants in each droplet's lifetime when it attained a diameter of 150, 100, and 50 μ, respectively.

Data (Fig. 3) show that the ratio r_c/r_l increases in magnitude as the droplet is consumed. This observed change in r_c/r_l during combustion of a droplet may be interpreted by means of fluid dynamic analysis of the flow associated with the burning of a fuel droplet.[13] A fuel droplet, burning under conditions where convective flow obtains, may be treated hydrodynamically as a point source discharging radially into a gas stream moving with a velocity u. Under these conditions a contact surface will form. This contact surface is defined by the streamlines that separate the oxidizer gases from the combustion products emerging from the enclosed droplet flame. Mass transport across the contact surface may occur by diffusion but not by convection. Because of this fact, the location of the flame within the contact surface probably is determined by the position of the contact surface. Ideally, this contact surface attains a width equal to four times the distance from the center of the source (the droplet) to its forward stagnation point r_h.[13] The distance r_h is related to the radius of the source r_l, the velocity of the stream u, and the "strength" of the source $\epsilon\rho$ by the equation[13]

$$r_h = (r_l \epsilon \rho / 8uk)^{1/2} \tag{5}$$

where k is the density of the stream gas. When the source is diminishing in size with time at a rate given by $D^2 = D_0^2 - \epsilon t$, the ratio of source radius r_l to contact surface radius r_h becomes

$$r_l/r_h = (8ukr_0/\rho\epsilon)^{1/2}(1 - t/\tau)^{1/4} \tag{6}$$

where r_0 is the initial radius of the droplet, and τ is the lifetime of the droplet, $\tau = D_0^2/\epsilon$.

If it is assumed that the maximum width of the droplet flame is proportional to the width of the contact surface, then the observed ratio of droplet radius to flame diameter should be proportional to the time factor $(1 - t/\tau)^{1/4}$ shown in Eq. (6). The data presented in Fig. 5 exhibit this behavior and suggest that the flame dimensions are determined largely by the contact surface dimensions.

Because the contact surface defines the zone of influence of a burning droplet, the overlapping of individual contact surfaces of droplets burning in sprays may affect profoundly the overall burning rate of the fuel spray. Experimental

Table 3 Effect of oxidizer composition Y on ratio of combustion zone radius to droplet radius r_c/r_l in burning of hexadecane droplets

Droplet diameter	$(r_c/r_l)_{Y=0.24}$ (droplet burning in air)		$(r_c/r_l)_{Y=x}/(r_c/r_l)_{Y=0.24}$					
			$x = 0.20$		$x = 0.41$		$x = 0.81$	
D, μ	Expt.	Theory[a]	Expt.	Theory[a]	Expt.	Theory[a]	Expt.	Theory[a]
150	4.9	23.6	1.1	1.08	0.72	0.741	0.53	0.505
100	6.7	23.6	1.1	1.08	0.66	0.741	0.58	0.505
50	10.1	23.6	1.3	1.08	0.67	0.741	0.55	0.505

[a] The theoretical values were calculated from Eq. (4) using the values of transport and thermodynamic parameters specified in Fig. 4.

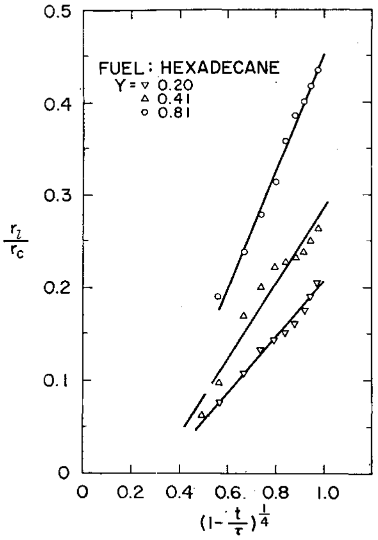

Fig. 5 Variation of r_l/r_c as a function of the $\frac{1}{4}$th power of the expended fractional lifetimes, t/τ, of burning droplets of hexadecane

and theoretical assessment of the role of such interactions in sprays of droplets in the size range of interest needs to be examined further.

Droplet Heat-Up and Mass Diffusion

Under certain conditions in oxygen-poor atmospheres, a charred, black residue remained after the liquid fuel in a no. 2 heating oil droplet had burned or vaporized completely. The appearance of such a deposit, which has been termed "coke," seemed to be associated with thermal decomposition of the fuel, and its formation, therefore, is dependent on the thermal and diffusional processes occurring in the liquid droplet. Internal circulation would tend to eliminate thermal and compositional variations in the liquid droplet, but it has been suggested[14] that such internal mixing, which has been observed[9] in large drops ($D > 1000\ \mu$), diminishes as drop size decreases. It is of interest, therefore, to examine in detail the heat-up and mass diffusional processes occurring in the liquid phase of a burning fuel droplet.

The process that sustains droplet combustion is the transport of heat from the surrounding flame to the droplet. At any given time in the course of burning, this heat serves both to evaporate fuel and to heat the interior of the droplet. The degree of internal heating depends in a complicated way on droplet size, time, thermal properties of the liquid, and rate of burning. A mathematical solution of the problem for pure fuel droplets without circulation in the liquid phase[15] is approximately valid for multicomponent fuels as well and may be used to estimate the degree of internal heating for the fuels employed in this study.

The important parameter in this problem is the dimensionless number b given by the expression

$$b = (\epsilon/8\alpha)^{1/2} \tag{7}$$

where α is the thermal diffusivity of the liquid fuel. A large value of b ($\gg 1$) corresponds to a situation in which internal heating is unimportant, whereas a value of the order of 1 or smaller corresponds to a situation in which internal heating is important. The internal temperature-time history of a burning droplet depends both on the parameter b [Eq. (7)] and on the surface temperature of the droplet. In the case of pure fuels, theory predicts[16] and measurements have shown[9] that the surface temperature is a few tens of degrees less than the boiling point. For multicomponent fuels, the boiling point is not defined well. The surface temperature, however, can be assumed to be approximately equal to that temperature at which 50% of the fuel oil can be removed by distillation at 1 atm total pressure. For no. 2 heating oil burning at 1 atm, this corresponds to a surface temperature of about 525°K. As long as diffusion within the liquid is confined to a thin surface layer, the surface temperature will not change significantly with time. Consequently, the cited surface temperature is also the maximum temperature that the liquids can attain.

In the case of no. 2 heating oil droplets, experimental measurements of burning rate correspond to b values [Eq. (7)] in the range 0.9 to 1.7. For the cited range of values, internal heating is substantial although differing from one end of the range to the other. As an example, the center of a droplet, burning at a rate such that $b = 1$, will reach a temperature of 500°K during a half-life. For $b = 1.5$, the center temperature would reach only 390°K, and for $b = 2$ only 330° K. Internal temperatures for other conditions can be derived from the theoretical solution.[15]

Multicomponent fuels such as no. 2 heating oil differ from pure fuels in that composition changes by distillation can occur during the lifetime of a burning multicomponent fuel droplet. The burning rate may not be affected significantly by such composition changes, but the tendency to form solid coke will be. At the surface of a fuel oil droplet that has just begun to burn, low-boiling components will be evaporated preferentially, thereby establishing concentration gradients within the droplet. The thickness of the affected zone will depend on the ability of light components to diffuse from the interior of the droplet toward the surface and on the rate at which the size of the droplet is decreasing. Estimates of the rates of diffusion for various possible conditions indicate that diffusion will affect only an extremely thin surface layer during most of the lifetime of a burning fuel oil droplet but may affect the entire droplet in the last fraction of the droplet lifetime. This comes about primarily because the diffusion constants for liquids are strongly dependent on temperature and therefore on the internal heating of the droplet. Consequently the degree of internal heating determines the time in a burning droplet's lifetime when diffusion becomes an important process.

When the droplet size becomes sufficiently small, diffusion of volatile components from the center region of the droplet will result first in a depletion of the volatile components and second in a rise of the surface temperature from the value just cited, $T_l \approx 525$°K, to a value slightly less than the boiling point of the least volatile component of the heating oil, that is, to a value about 575°K or perhaps slightly higher.

Coking, if it occurs, probably would occur at this time when both the surface and interior temperatures have reached their maximum values. Because coking depends on both the time of exposure to high temperature and on the temperature itself, the critical conditions cannot be predicted. However, coking will be favored by these circumstances: 1) a low b value [Eq. (7)], 2) large initial droplet diameter, and 3) the presence of high molecular weight components in the fuel oil.

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Free-Molecule Flow and Convective-Radiative Energy Transport in a Tapered Tube or Conical Nozzle

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An analysis is presented of the fluid flow and heat transfer characteristics of a highly rarefied gas passing through a tapered tube or conical nozzle. The rate of mass throughflow has been determined as a function of the pressures and temperatures of the system and of the tube dimensions. It is found that, at moderate and large angles of taper, the mass throughflow is affected little by increases in tube length, except for short tubes. At small taper angles, the mass throughflow is more sensitive to tube length. The energy transport analysis includes simultaneous convection and radiation. Numerical results have been found for the adiabatic wall temperature. It is found that, for surfaces that can be approximated realistically as diffuse emitters and reflectors of thermal radiation, the results differ little from those for pure radiation.

Nomenclature

A = surface area
A_1 = tube cross section, small end
A_2 = tube cross section, large end
a = accommodation coefficient
e = convective energy/time-area
F = angle factor
f = solution of Eq. (3)
g = solution of Eq. (5)
L = tube length
M = rate of mass throughflow
m = mass flux/time-area
p = pressure
q = local heat flux/time-area
R = gas constant
r_1 = tube radius, small end
r_2 = tube radius, large end
T = absolute temperature
x = axial coordinate
α = absorptivity
β = ratio of convective to radiative efflux
γ = specific heat ratio
ϵ = emissivity
θ = half-taper angle
Λ = vertex to entrance distance
ξ = axial distance from vertex
σ = Stefan-Boltzmann constant
τ = temperature variable

Subscripts

1 = chamber 1
2 = chamber 2
r,rad = radiative
c = convective

Introduction

The internal flow of rarefied gases has been a subject of study for over half a century. Almost all of the work has been concerned with parallel-walled conduits such as the circular tube and the flat rectangular duct. Only very recently has attention been directed to less elementary passage shapes, for instance, Refs. 1 and 2. Of particular interest among these is the tapered tube or conical nozzle, which includes the circular tube as a special case.

The paper is concerned with both the fluid flow and the heat transfer characteristics of a highly rarefied gas passing through a tapered tube or conical nozzle. The density level is such that collisions among molecules of the gas are much less probable than are collisions between the gas molecules

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